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Extended hamiltonian action for arbitrary spin fields in flat and AdS spaces

R.R. Metsaev*

Department of Theoretical Physics, P.N. Lebedev Physical Institute,
Leninsky prospect 53, Moscow 119991, Russia

Abstract

Totally symmetric arbitrary spin massless and massive free fields in flat and AdS spaces are studied. Extended gauge invariant hamiltonian action for such fields is obtained. The action is constructed out of phase space fields and Lagrange multipliers which are free of algebraic constraints. Gauge transformations of the phase space fields and Lagrange multipliers are derived. Lie derivative realization of flat and AdS space-time symmetries on the phase space fields and Lagrange multipliers is obtained. Canonical realization of space-time symmetries is also found. Use of the Poincaré parametrization of AdS space allows us to treat fields in flat space and AdS space on equal footing.

* E-mail: metsaev@lpi.ru

1 Introduction

In view of the aesthetic features of extended hamiltonian approach to the relativistic field dynamics a interest in this approach was periodically renewed (see e.g. Refs.[1, 2]). The extended hamiltonian approach provides systematic and self-contained way to study many aspects of relativistic field dynamics. Progress in understanding higher-spin field dynamics [3] has lead to intensive and in-depth study of various aspects of AdS field dynamics. Lagrangian formulation of higher-spin fields was developed many years ago in Refs.[4, 5]. By now many interesting approaches to AdS fields are known in the literature. However we note that the extended hamiltonian formulation of massless and massive higher-spin fields in flat and AdS spaces of arbitrary dimensions has not yet been worked out.[1]

The purpose of this paper is to develop gauge invariant hamiltonian approach to totally symmetric arbitrary spin massless and massive fields in flat and AdS spaces. In this paper we deal with free bosonic fields. Our approach to the extended hamiltonian field dynamics can be summarized as follows.

i) We start with Lagrangian formulation of massless and massive fields in flat and AdS spaces and use representation for Lagrangian in terms of de Donder like divergence obtained in Refs.[9, 10, 11]. We consider fields in d dimensional flat space and $d+1$ dimensional AdS space. We use the Poincaré parametrization of AdS_{d+1} space in which the Lorentz algebra $so(d-1,1)$ symmetries are realized manifestly. We use the double-traceless higher-spin fields of the Lorentz algebra $so(d-1,1)$. It is the use of such double-traceless fields and the Poincaré parametrization of AdS space that allows us to treat massless and massive fields in flat and AdS spaces on equal footing.

ii) Our extended hamiltonian action is formulated in terms of $so(d-1)$ algebra fields. All fields appearing in our extended hamiltonian formulation are free of algebraic constraints. Field content entering our extended hamiltonian action involves phase space fields and Lagrange multipliers. Number of the Lagrangian multipliers and half of the phase space fields is equal to the number of gauge fields appearing in the Lagrangian formulation.

Our paper is organized as follows.

In section 2, we review the Lagrangian formulation of massless and massive fields in flat and AdS spaces. We discuss representation for Lagrangian in terms of the de Donder like divergence found in Refs.[9, 10, 11]. We review gauge symmetries of the Lagrangian and realization of space-time symmetries on the space of gauge fields.

Sec. 3 is devoted to extended hamiltonian formulation of massless and massive fields in flat and AdS spaces. We start with description of field content appearing in our approach. After this we present our result for extended hamiltonian action and the corresponding gauge transformations.

In section 4, we discuss realization of space-time symmetries in the framework of extended hamiltonian approach. This is to say that we discuss realization of the Poincaré symmetries on space of gauge fields in flat space and realization of the $so(d,2)$ algebra symmetries on space of gauge fields in AdS_{d+1} space. We present the Lie derivative realization and the canonical realization of flat and AdS space-time symmetries on the phase space fields and Lagrange multipliers entering the extended hamiltonian approach.

In Appendix, we summarize our conventions and the notation.

[1] Discussion of hamiltonian formulation of massless fermionic fields in $4d$ flat and AdS_4 spaces may be found in Refs.[6, 7] (for some discussion of massless bosonic fields in AdS_4 see Ref.[7]). Discussion of hamiltonian formulation of massive spin 3/2 fermionic field in $4d$ flat space may be found in Ref.[8].

2 Gauge invariant Lagrangian via de Donder like divergence

In metric like approach, gauge invariant Lagrangian for free massless fields in flat and AdS_4 spaces was obtained in Refs.[4, 5], while gauge invariant Lagrangian for free massive fields in flat and AdS_{d+1} spaces was found in Ref.[12].[2] In Refs.[9, 10, 11], we noticed that use of de Donder like divergence simplifies considerably the structure of gauge invariant Lagrangian. Representation of the gauge invariant Lagrangian for massive field in flat space in terms of modified de Donder divergence was obtained in Ref.[9], while representation of the gauge invariant Lagrangian for massless and massive fields in AdS space in terms of the modified de Donder divergence was found in Refs.[10, 11]. Because representation of the gauge invariant Lagrangian via the modified de Donder divergence turns out to be helpful for the derivation of extended hamiltonian action we start with review of our results in Refs.[9, 10, 11]. Before proceeding to the review we note that we use the Cartesian parametrization of Minkowski space and the Poincaré parametrization of AdS_{d+1} space (for the notation, see Appendix A),

$$ds^2 = dx^a dx^a\,, \qquad\qquad \text{for flat space}, \tag{2.1}$$

$$ds^2 = \frac{1}{z^2}(dx^a dx^a + dzdz)\,, \qquad\qquad \text{for AdS space}. \tag{2.2}$$

The use of such parametrizations allows us, among other thing, to treat fields in flat and AdS spaces on equal footing. We now begin our review with the discussion of field contents.

Field content for massless field in $R^{d-1,1}$. As is well known [4], spin-s massless field in d-dimensional flat space can be described by the rank-s totally symmetric tensor field of the Lorentz algebra $so(d-1,1)$,

$$\phi^{a_1\ldots a_s}\,, \tag{2.3}$$

subject to the double-tracelessness constraint, $\phi^{aabba_5\ldots a_s} = 0$. To simplify the presentation of gauge invariant action we use oscillators α^a and introduce the following ket-vector:

$$|\phi\rangle \equiv \frac{1}{s!}\alpha^{a_1}\ldots\alpha^{a_s}\phi^{a_1\ldots a_s}|0\rangle\,. \tag{2.4}$$

Field content for massive field in $R^{d-1,1}$. As is well known [12], spin-s massive field in flat space can be described by the following set of fields

$$\phi^{a_1\ldots a_{s'}}\,, \qquad s' = 0,1,\ldots,s. \tag{2.5}$$

Fields in (2.5) with $s'=0$, $s'=1$, and $s'\geq 2$ are the respective scalar, vector, and rank-s' totally symmetric fields of the Lorentz algebra $so(d-1,1)$. Fields in (2.5) with $s'\geq 4$ are double-traceless, $\phi^{aabba_5\ldots a_{s'}} = 0$. To streamline the presentation of gauge invariant action we use oscillators α^a, ζ and introduce the following ket-vector:

$$|\phi\rangle \equiv \sum_{s'=0}^{s} \frac{\zeta^{s-s'}\alpha^{a_1}\ldots\alpha^{a_{s'}}}{s'!\sqrt{(s-s')!}}\,\phi^{a_1\ldots a_{s'}}|0\rangle\,. \tag{2.6}$$

[2] For arbitrary d, various gauge invariant formulations of massless fields in AdS_{d+1} were discussed in Refs.[13, 14, 15]. In earlier literature, study of arbitrary spin massive field in flat space via dimensional reduction may be found in Refs.[16, 17, 18]. Discussion of various dimensional reduction techniques in AdS may be found in Refs.[19, 20, 21]. In recent years, higher-spin gauge fields have also been extensively studied in the framework of BRST approach (see e.g. Refs.[22]-[25]). Frame-like approach to massive fields was developed in Refs.[26, 27]. In the framework of light-cone gauge, the higher-spin AdS fields were studied in Refs.[14, 28, 29].

Field content for massless field in AdS_{d+1}. To discuss gauge invariant formulation of spin-s massless field in AdS_{d+1} we use the following set of fields in Ref.[10]::

$$\boldsymbol{\phi}^{a_1\ldots a_{s'}}\,, \qquad s'=0,1,\ldots,s\,. \tag{2.7}$$

Fields in (2.7) with $s'=0$, $s'=1$, and $s'\geq 2$ are the respective scalar, vector, and rank-s' totally symmetric fields of the Lorentz algebra $so(d-1,1)$. Fields in (2.7) with $s'\geq 4$ are double-traceless, $\boldsymbol{\phi}^{aabba_5\ldots a_{s'}}=0$.[3] To discuss gauge invariant Lagrangian in easy-to-use form we use oscillators α^a, α^z to collect fields (2.7) into the ket-vector

$$|\boldsymbol{\phi}\rangle \equiv \sum_{s'=0}^{s} \frac{\alpha_z^{s-s'}\alpha^{a_1}\ldots\alpha^{a_{s'}}}{s'!\sqrt{(s-s')!}}\,\boldsymbol{\phi}^{a_1\ldots a_{s'}}|0\rangle\,. \tag{2.8}$$

Field content for massive field in AdS_{d+1}. To discuss gauge invariant formulation of spin-s massive field in AdS_{d+1} we use the following set of fields in Ref.[11]:

$$\boldsymbol{\phi}_n^{a_1\ldots a_{s'}}\,, \qquad n\in[s-s']_2\,, \qquad s'=0,1,\ldots,s-1,s\,, \tag{2.9}$$

(for notation, see (A.2) in Appendix). Fields in (2.9) with $s'=0$, $s'=1$, and $s'\geq 2$ are the respective scalar, vector, and rank-s' totally symmetric fields of the Lorentz algebra $so(d-1,1)$. Fields in (2.9) with $s'\geq 4$ are double-traceless, $\boldsymbol{\phi}_n^{aabba_5\ldots a_{s'}}=0$.[4] To streamline the presentation we use oscillators α^a, α^z, ζ and collect fields (2.9) into the ket-vector defined by

$$|\boldsymbol{\phi}\rangle = \sum_{s'=0}^{s}\sum_{n\in[s-s']_2} \frac{\zeta^{\frac{s-s'+n}{2}}\alpha_z^{\frac{s-s'-n}{2}}\alpha^{a_1}\ldots\alpha^{a_{s'}}}{s'!\sqrt{(\frac{s-s'+n}{2})!(\frac{s-s'-n}{2})!}}\,\boldsymbol{\phi}_n^{a_1\ldots a_{s'}}|0\rangle\,. \tag{2.10}$$

Lagrangian. Gauge invariant action for fields in flat and AdS spaces is given by

$$S=\int d^dx\,\mathcal{L}\,, \qquad\qquad \text{for flat space}, \tag{2.11}$$

$$S=\int d^dx\,dz\,\mathcal{L}\,, \qquad\qquad \text{for AdS space}, \tag{2.12}$$

where Lagrangian we found is given by

$$\mathcal{L} = -\frac{1}{2}\langle\partial^a\boldsymbol{\phi}|\boldsymbol{\mu}|\partial^a\boldsymbol{\phi}\rangle - \frac{1}{2}\langle\boldsymbol{\phi}|\boldsymbol{\mu}\mathcal{M}^2|\boldsymbol{\phi}\rangle + \frac{1}{2}\langle\bar{\mathbf{C}}\boldsymbol{\phi}||\bar{\mathbf{C}}\boldsymbol{\phi}\rangle\,, \tag{2.13}$$

$$\bar{\mathbf{C}} \equiv \bar{\boldsymbol{\alpha}}\boldsymbol{\partial} - \frac{1}{2}\boldsymbol{\alpha}\boldsymbol{\partial}\bar{\boldsymbol{\alpha}}^2 - \bar{e}_1\boldsymbol{\Pi}^{[1,2]} + \frac{1}{2}e_1\bar{\boldsymbol{\alpha}}^2\,, \tag{2.14}$$

$$\mathbf{C} \equiv \boldsymbol{\alpha}\boldsymbol{\partial} - \frac{1}{2}\boldsymbol{\alpha}^2\bar{\boldsymbol{\alpha}}\boldsymbol{\partial} - e_1\boldsymbol{\Pi}^{[1,2]} + \frac{1}{2}\bar{e}_1\boldsymbol{\alpha}^2\,, \tag{2.15}$$

[3] In Ref.[5], the spin-s massless field in AdS_{d+1} is described by rank-s totally symmetric doubletraceless tensor field of the Lorentz algebra $so(d,1)$. Note that $so(d-1,1)$ tensorial components of the tensor field in Ref.[5] are not double-traceless. The tensor field in Ref.[5] is related to our fields (2.7) by invertible transformation. This invertible transformation is described in Ref.[10].

[4] In Ref.[12], the spin-s massive field in AdS_{d+1} is described by the set of fields involving totally symmetric doubletraceless tensor fields of the Lorentz algebra $so(d,1)$. Note that $so(d-1,1)$ tensorial components of the tensor fields in Ref.[12] are not double-traceless. The fields in Ref.[12] are related to our fields (2.9) by invertible transformation. This invertible transformation is described in Ref.[11].

$$\boldsymbol{\mu} \equiv 1 - \frac{1}{4}\boldsymbol{\alpha}^2\bar{\boldsymbol{\alpha}}^2 , \qquad \boldsymbol{\Pi}^{[1,2]} \equiv 1 - \boldsymbol{\alpha}^2 \frac{1}{2(2N_{\boldsymbol{\alpha}} + d)}\bar{\boldsymbol{\alpha}}^2 , \tag{2.16}$$

and expressions like $\boldsymbol{\alpha\partial}$, $\boldsymbol{\alpha}^2$ are defined in Appendix (see (A.6), (A.7)). We note that the Lagrangian for massless and massive fields in flat and AdS spaces is distinguished only by the operators $\mathcal{M}^2$, e_1 and $\bar{e}_1$. To see this, we now present the explicit form of these operators in turn.

Operators $\mathcal{M}^2$, e_1, $\bar{e}_1$ for massless field in flat space:

$$\mathcal{M}^2 = 0 , \qquad e_1 = 0 , \qquad \bar{e}_1 = 0 . \tag{2.17}$$

Operators $\mathcal{M}^2$, e_1, $\bar{e}_1$ for massive field in flat space:

$$\mathcal{M}^2 = m^2 , \tag{2.18}$$

$$e_1 = m\zeta\widetilde{e}_1 , \qquad \bar{e}_1 = -m\widetilde{e}_1\bar{\zeta} , \tag{2.19}$$

$$\widetilde{e}_1 \equiv \Big(\frac{2s + d - 4 - N_\zeta}{2s + d - 4 - 2N_\zeta}\Big)^{1/2} . \tag{2.20}$$

In (2.18),(2.19), m stands for the commonly used mass parameter of the massive field.

Operators $\mathcal{M}^2$, e_1, $\bar{e}_1$ for massless field in AdS_{d+1}:

$$\mathcal{M}^2 = -\partial_z^2 + \frac{1}{z^2}(\nu^2 - \frac{1}{4}) , \tag{2.21}$$

$$-e_1 = \alpha^z r_z \mathcal{T}_{\nu-\frac{1}{2}} , \qquad -\bar{e}_1 = \mathcal{T}_{-\nu+\frac{1}{2}} r_z \bar{\alpha}^z , \tag{2.22}$$

$$\mathcal{T}_\nu \equiv \partial_z + \frac{\nu}{z} , \qquad \nu \equiv s + \frac{d-4}{2} - N_z , \tag{2.23}$$

$$r_z \equiv \Big(\frac{2s + d - 4 - N_z}{2s + d - 4 - 2N_z}\Big)^{1/2} . \tag{2.24}$$

Operators $\mathcal{M}^2$, e_1, $\bar{e}_1$ for massive field in AdS_{d+1}:

$$\mathcal{M}^2 \equiv -\partial_z^2 + \frac{1}{z^2}(\nu^2 - \frac{1}{4}) , \tag{2.25}$$

$$-e_1 = \zeta r_\zeta \mathcal{T}_{-\nu-\frac{1}{2}} + \alpha^z r_z \mathcal{T}_{\nu-\frac{1}{2}} , \qquad -\bar{e}_1 = \mathcal{T}_{\nu+\frac{1}{2}} r_\zeta \bar{\zeta} + \mathcal{T}_{-\nu+\frac{1}{2}} r_z \bar{\alpha}^z , \tag{2.26}$$

$$\mathcal{T}_\nu \equiv \partial_z + \frac{\nu}{z} , \qquad \nu \equiv \kappa + N_\zeta - N_z , \tag{2.27}$$

$$r_\zeta = \left(\frac{(s + \frac{d-4}{2} - N_\zeta)(\kappa - s - \frac{d-4}{2} + N_\zeta)(\kappa + 1 + N_\zeta)}{2(s + \frac{d-4}{2} - N_\zeta - N_z)(\kappa + N_\zeta - N_z)(\kappa + N_\zeta - N_z + 1)}\right)^{1/2} , \tag{2.28}$$

$$r_z = \left(\frac{(s + \frac{d-4}{2} - N_z)(\kappa + s + \frac{d-4}{2} - N_z)(\kappa - 1 - N_z)}{2(s + \frac{d-4}{2} - N_\zeta - N_z)(\kappa + N_\zeta - N_z)(\kappa + N_\zeta - N_z - 1)}\right)^{1/2} , \tag{2.29}$$

$$\kappa \equiv \sqrt{m^2 + \Big(s + \frac{d-4}{2}\Big)^2} . \tag{2.30}$$

In (2.30), m stands for the commonly used mass parameter of the spin-s massive field in AdS_{d+1}.

The following remarks are in order.

i) We note that it is the quantity $\bar{\mathbf{C}}|\phi\rangle$ that we refer to as de Donder like divergence (or modified de Donder divergence). Only for the case of massless field in flat space this de Donder like divergence coincides with the standard well-known de Donder divergence. From (2.13),(2.14), we see that many complicated terms contributing to the Lagrangian are collected into the de Donder like divergence. Thus, as we have promised, use of the de Donder like divergence allows us to simplify considerably a structure of the Lagrangian.[5]

ii) If we represent the $\langle\bar{\mathbf{C}}\phi|\bar{\mathbf{C}}\phi\rangle$ contribution to the Lagrangian in terms of the derivatives and oscillators, then, for the case of massless and massive fields in flat space, Lagrangian given in (2.13) takes the same form as in Refs.[4, 12]. For the case of massless and massive fields in AdS_{d+1}, in order to cast the Lagrangians in Refs.[4, 12] into the form given in (2.13) we use our set of the $so(d-1,1)$ algebra double -traceless gauge fields. We recall that, in Refs.[4, 12], the Lagrangians of massless and massive fields in AdS_{d+1} are formulated in terms of $so(d,1)$ algebra double-traceless gauge fields. Our gauge fields are related to gauge fields used in Refs.[4, 12] by invertible transformations. The invertible transformations are described in Refs.[10, 11].

iii) We note that, representation for Lagrangian in (2.13) -(2.16) is universal and is valid for arbitrary Poincaré invariant theory. Various Poincaré invariant theories are distinguished by the operators $\mathcal{M}^2$, e_1, $\bar{e}_1$. This to say, that the dependence of the operators $\mathbf{C}$, $\bar{\mathbf{C}}$ on the oscillators α^a, $\bar{\alpha}^a$ and the flat derivative ∂^a takes the same form for massless and massive fields in flat and AdS spaces. In other words, the operators $\mathbf{C}$, $\bar{\mathbf{C}}$ for massless and massive fields in flat and AdS spaces are distinguished only by the operators e_1 and $\bar{e}_1$. A a side remark we note that, using auxiliary fields and Stueckelberg fields, Lagrangian for totally symmetric arbitrary spin conformal fields can also be cast into the form given in (2.13) (see Refs.[35, 36]).

iv) Representation for Lagrangian given in (2.13) turns out to be especially helpful for the study of AdS/CFT duality for arbitrary spin massless and massive bulk AdS fields and the corresponding boundary current and shadow fields (see Refs.[37]-[39].)

Gauge symmetries. We now discuss gauge symmetries of Lagrangian given in (2.13). We begin with the description of gauge transformation parameters involved in gauge transformations of gauge fields. We discuss the gauge transformation parameters in turn.

Gauge transformations parameter for massless field in $R^{d-1,1}$. To discuss gauge symmetries of spin-s massless field in flat space we use the well-known gauge transformation parameter

$$\boldsymbol{\xi}^{a_1\ldots a_{s-1}}\,, \tag{2.31}$$

which is rank-$(s-1)$ totally symmetric tensor field of the Lorentz algebra $so(d-1,1)$. For $s\geq 3$ this parameter is traceless, $\boldsymbol{\xi}^{aaa_3\ldots a_{s-1}}=0$ (see Ref.[4]). To simplify the presentation we use the oscillator α^a and introduce the ket-vector

$$|\boldsymbol{\xi}\rangle \equiv \frac{1}{(s-1)!}\alpha^{a_1}\ldots\alpha^{a_{s-1}}\boldsymbol{\xi}^{a_1\ldots a_{s-1}}|0\rangle\,. \tag{2.32}$$

Gauge transformations parameters for massive field in $R^{d-1,1}$. Gauge symmetries of spin-s massive field in flat space are described by the following set of gauge transformations parameters in Ref.[12]:

$$\boldsymbol{\xi}^{a_1\ldots a_{s'}}\,,\qquad s'=0,1,\ldots,s-1. \tag{2.33}$$

[5] Because our modified de Donder gauge leads to considerably simplified analysis of AdS field dynamics we believe that this gauge might also be useful for better understanding of various aspects of AdS/QCD correspondence which are discussed e.g. in Ref.[31]. Interesting applications of the *standard* de Donder-Feynman gauge to the various problems of higher-spin fields may be found in Refs.[32, 33, 34].

Gauge transformation parameters in (2.33) with $s' = 0$, $s' = 1$, and $s' \geq 2$ are the respective scalar, vector, and rank-s' totally symmetric fields of the Lorentz algebra $so(d-1,1)$. Gauge transformation parameters in (2.33) with $s' \geq 2$ are traceless, $\boldsymbol{\xi}^{aaa_3\ldots a_{s'}} = 0$. To streamline the presentation we use the oscillators α^a, ζ and introduce the following ket-vector:

$$|\boldsymbol{\xi}\rangle \equiv \sum_{s'=0}^{s-1} \frac{\zeta^{s-1-s'}\alpha^{a_1}\ldots\alpha^{a_{s'}}}{s'!\sqrt{(s-1-s')!}}\,\boldsymbol{\xi}^{a_1\ldots a_{s'}}|0\rangle\,. \tag{2.34}$$

Gauge transformations parameters for massless field in AdS_{d+1}. To discuss gauge symmetries of spin-s massless field in AdS_{d+1} we use the following set of gauge transformation parameters in Ref.[10]:

$$\boldsymbol{\xi}^{a_1\ldots a_{s'}}\,, \qquad s' = 0,1,\ldots,s-1\,. \tag{2.35}$$

Gauge transformation parameters in (2.35) with $s' = 0$, $s' = 1$, and $s' \geq 2$ are the respective scalar, vector, and rank-s' totally symmetric fields of the Lorentz algebra $so(d-1,1)$. The gauge transformation parameters in (2.35) with $s' \geq 2$ are traceless, $\boldsymbol{\xi}^{aaa_3\ldots a_{s'}} = 0$.[6] To simplify the presentation we use the oscillators α^a, α^z and collect gauge transformation parameters (2.35) into the ket-vector given by

$$|\boldsymbol{\xi}\rangle \equiv \sum_{s'=0}^{s-1} \frac{\alpha_z^{s-1-s'}\alpha^{a_1}\ldots\alpha^{a_{s'}}}{s'!\sqrt{(s-1-s')!}}\,\boldsymbol{\xi}^{a_1\ldots a_{s'}}|0\rangle\,. \tag{2.36}$$

Gauge transformations parameters for massive field in AdS_{d+1}. To describe gauge symmetries of spin-s massive field in AdS_{d+1} we use the following set of gauge transformation parameters in Ref.[11]:

$$\boldsymbol{\xi}_n^{a_1\ldots a_{s'}}\,, \qquad n \in [s-1-s']_2\,, \qquad s' = 0,1,\ldots,s-1\,. \tag{2.37}$$

Gauge transformation parameters in (2.37) with $s' = 0$, $s' = 1$, and $s' \geq 2$ are the respective scalar, vector, and rank-s' totally symmetric fields of the Lorentz algebra $so(d-1,1)$. The gauge transformation parameters in (2.37) with $s' \geq 2$ are traceless, $\boldsymbol{\xi}_n^{aaa_3\ldots a_{s'}} = 0$.[7] To streamline the presentation of gauge invariant Lagrangian we use oscillators α^a, α_z, ζ and collect gauge transformation parameters (2.37) into the ket-vector defined by

$$|\boldsymbol{\xi}\rangle = \sum_{s'=0}^{s-1}\ \sum_{n\in[s-1-s']_2} \frac{\zeta^{\frac{s-1-s'+n}{2}}\alpha_z^{\frac{s-1-s'-n}{2}}\alpha^{a_1}\ldots\alpha^{a_{s'}}}{s'!\sqrt{(\frac{s-1-s'+n}{2})!(\frac{s-1-s'-n}{2})!}}\,\boldsymbol{\xi}_n^{a_1\ldots a_{s'}}|0\rangle\,. \tag{2.38}$$

Having represented the field contents and gauge transformation parameters in terms of the ket-vectors $|\boldsymbol{\phi}\rangle$ and $|\boldsymbol{\xi}\rangle$ we note that the gauge transformations can entirely be presented in terms of these ket-vectors. The representation for gauge transformations found in Refs.[9, 10, 11] is given by

$$\delta|\boldsymbol{\phi}\rangle = G|\boldsymbol{\xi}\rangle\,, \qquad G \equiv \boldsymbol{\alpha\partial} - e_1 - \boldsymbol{\alpha}^2\frac{1}{2N_{\boldsymbol{\alpha}}+d-2}\bar{e}_1\,. \tag{2.39}$$

[6] In Ref.[5], gauge symmetries of spin-s massless field in AdS_{d+1} are described by gauge transformation parameter which is rank-$(s-1)$ totally symmetric traceless tensor field of the Lorentz algebra $so(d,1)$. Note that $so(d-1,1)$ tensorial components of this gauge transformation parameter are not traceless. Gauge transformation parameter in Ref.[5] is related to our gauge transformation parameters (2.35) by invertible transformation described in Ref.[10].

[7] In Ref.[12], gauge symmetries of spin-s massive field in AdS_{d+1} are described by gauge transformation parameters which are totally symmetric traceless tensor fields of the Lorentz algebra $so(d,1)$. The $so(d-1,1)$ tensorial components of these gauge transformation parameters are not traceless. Gauge transformation parameters in Ref.[12] are related to our gauge transformation parameters (2.37) by invertible transformation described in Ref.[11].

For the cases of massless and massive fields in flat space, gauge transformations (2.39) coincide with the respective gauge transformations found in Ref.[4] and Ref.[12]. For the case of massless and massive fields in AdS_{d+1}, in order to cast the gauge transformations in Refs.[4, 12] into the form given in (2.39) we use our set of the $so(d-1,1)$ algebra double -traceless gauge fields and $so(d-1,1)$ algebra traceless gauge transformation parameters. We recall that, in Refs.[4, 12], gauge transformations of massless and massive fields in AdS_{d+1} are formulated in terms of $so(d,1)$ algebra double-traceless gauge fields and the $so(d,1)$ algebra traceless gauge transformation parameters. Our fields and gauge transforation parameters are related to fields and gauge transformation parameters used in Refs.[4, 12] by invertible transformations. The invertible transformations are described in Refs.[10, 11].

Space-time symmetries. We now discuss realization of space-time symmetries. For the case of fields in d-dimensional flat space, the space-time symmetries are described by the Poincaré algebra which consist translation generators P^a and the $so(d-1,1)$ generators of the Lorentz algebra J^{ab}. For the case of fields in AdS_{d+1}, the space-time symmetries are described by the $so(d,2)$ algebra. Using the Poincaré parametrization of AdS_{d+1} space, we note that $so(d,2)$ algebra in the basis of $so(d-1,1)$ algebra consists of translation generators P^a, dilatation generator D, conformal boost generators K^a, and generators of the $so(d-1,1)$ Lorentz algebra J^{ab}. To summarize, space-time symmetries of fields are described by the following generators:[8]

$$P^a\,, \quad J^{ab}\,, \qquad\qquad \text{for fields in } R^{d-1,1} \tag{2.40}$$

$$P^a\,, \quad J^{ab}\,, \quad D\,, \quad K^a\,, \qquad\qquad \text{for fields in } AdS_{d+1}\,. \tag{2.41}$$

We assume the following normalization for generators in (2.40),(2.41):

$$[D,P^a] = -P^a\,, \qquad\qquad [P^a,J^{bc}] = \eta^{ab}P^c - \eta^{ac}P^b, \tag{2.42}$$

$$[D,K^a] = K^a\,, \qquad\qquad [K^a,J^{bc}] = \eta^{ab}K^c - \eta^{ac}K^b, \tag{2.43}$$

$$[P^a,K^b] = \eta^{ab}D - J^{ab}\,, \tag{2.44}$$

$$[J^{ab},J^{ce}] = \eta^{bc}J^{ae} + 3 \text{ terms}\,. \tag{2.45}$$

The well-known realization of the generators P^a and J^{ab} takes the same form for both fields in flat space and fields in AdS space,

$$P^a = \partial^a\,, \tag{2.46}$$

$$J^{ab} = x^a\partial^b - x^b\partial^a + M^{ab}\,, \qquad\qquad M^{ab} = \alpha^a\bar{\alpha}^b - \alpha^b\bar{\alpha}^a\,. \tag{2.47}$$

For the case of fields in AdS space, to complete description of space-time symmetries we should find realization of generators D and K^a on space of AdS fields. The realization was found in Refs.[10, 11] and is given by

$$D = x^a\partial^a + \Delta\,, \qquad \Delta \equiv z\partial_z + \frac{d-1}{2}\,, \tag{2.48}$$

$$K^a = -\frac{1}{2}x^bx^b\partial^a + x^aD + M^{ab}x^b + R^a\,, \tag{2.49}$$

[8] In our approach, only $so(d-1,1)$ symmetries are realized manifestly. The $so(d,2)$ symmetries of fields in AdS_{d+1} could be realized manifestly by using ambient space approach (see e.g. Refs.[40]-[43].)

$$R^a = zr\bar{\alpha}^a + z\widetilde{I}^a\bar{r} - \frac{1}{2}z^2\partial^a\,, \qquad (2.50)$$

$$\widetilde{I}^a \equiv \alpha^a - \boldsymbol{\alpha}^2\frac{1}{2N_{\boldsymbol{\alpha}} + d - 2}\bar{\alpha}^a\,. \qquad (2.51)$$

Operators r, $\bar{r}$ appearing in (2.50) are given by

$$r \equiv -\alpha^z r_z\,, \qquad \bar{r} \equiv r_z\bar{\alpha}^z\,, \qquad \text{for massless AdS field;} \qquad (2.52)$$

$$r \equiv -\zeta r_\zeta - \alpha^z r_z\,, \qquad \bar{r} \equiv r_\zeta\bar{\zeta} + r_z\bar{\alpha}^z\,, \qquad \text{for massive AdS field}\,, \qquad (2.53)$$

where operator r_z for massless field is given in (2.24), while the operators r_ζ, r_z for massive field are given in (2.28), (2.29).

3 Extended gauge invariant hamiltonian action

We now discuss the extended gauge invariant hamiltonian action for massless and massive fields in flat and AdS spaces. We begin our discussion with the description of field contents. We discuss the field contents in turn.

Field content for massless field in $R^{d-1,1}$. To discuss hamiltonian action for spin-s massless field in flat space we introduce the following set of fields:

$$\phi^{i_1\ldots i_s}\,, \qquad \mathcal{P}^{i_1\ldots i_s}\,, \qquad (3.1)$$

$$\phi^{i_1\ldots i_{s-3}}\,, \qquad \mathcal{P}^{i_1\ldots i_{s-3}}\,, \qquad (3.2)$$

$$\lambda^{i_1\ldots i_{s-1}}\,, \qquad \lambda^{i_1\ldots i_{s-2}}\,. \qquad (3.3)$$

Fields in (3.1)-(3.3) are totally symmetric *traceful* tensor fields of the $so(d-1)$ algebra. Thus, we see that all our fields in (3.1)-(3.3) are free from any constraints, i.e. we deal with unconstrained fields. We note that fields in (3.1), (3.2) are phase space variables, while fields in (3.3) are Lagrange multipliers. The fields $\phi^{i_1\ldots i_s}$, $\phi^{i_1\ldots i_{s-3}}$ and Lagrange multipliers (3.3) are related to field in (2.3) by invertible transformation. To simplify the presentation we use oscillators α^i and collect fields (3.1)-(3.3) into the following ket-vectors:

$$|\phi_s\rangle \equiv \frac{1}{s!}\alpha^{i_1}\ldots\alpha^{i_s}\phi^{i_1\ldots i_s}|0\rangle\,, \qquad (3.4)$$

$$|\phi_{s-3}\rangle \equiv \frac{1}{(s-3)!}\alpha^{i_1}\ldots\alpha^{i_{s-3}}\phi^{i_1\ldots i_{s-3}}|0\rangle\,, \qquad (3.5)$$

$$|\mathcal{P}_s\rangle \equiv \frac{1}{s!}\alpha^{i_1}\ldots\alpha^{i_s}\mathcal{P}^{i_1\ldots i_s}|0\rangle\,, \qquad (3.6)$$

$$|\mathcal{P}_{s-3}\rangle \equiv \frac{1}{(s-3)!}\alpha^{i_1}\ldots\alpha^{i_{s-3}}\mathcal{P}^{i_1\ldots i_{s-3}}|0\rangle\,, \qquad (3.7)$$

$$|\lambda_{s-1}\rangle \equiv \frac{1}{(s-1)!}\alpha^{i_1}\ldots\alpha^{i_{s-1}}\lambda^{i_1\ldots i_{s-1}}|0\rangle\,, \qquad (3.8)$$

$$|\lambda_{s-2}\rangle \equiv \frac{1}{(s-2)!}\alpha^{i_1}\ldots\alpha^{i_{s-2}}\lambda^{i_1\ldots i_{s-2}}|0\rangle\,. \qquad (3.9)$$

Field content for massive field in $R^{d-1,1}$. To develop hamiltonian approach to spin-s massive field in flat space we introduce the following set of fields:

$$\phi_s^{i_1\ldots i_{s'}}\,, \qquad \mathcal{P}_s^{i_1\ldots i_{s'}}\,, \qquad s'=0,1,\ldots,s\,, \tag{3.10}$$

$$\phi_{s-3}^{i_1\ldots i_{s'}}\,, \qquad \mathcal{P}_{s-3}^{i_1\ldots i_{s'}}\,, \qquad s'=0,1,\ldots,s-3\,, \tag{3.11}$$

$$\lambda_{s-1}^{i_1\ldots i_{s'}}\,, \qquad s'=0,1,\ldots,s-1\,, \tag{3.12}$$

$$\lambda_{s-2}^{i_1\ldots i_{s'}}\,, \qquad s'=0,1,\ldots,s-2\,. \tag{3.13}$$

Fields in (3.10)-(3.13) with $s'=0$, $s'=1$, and $s'\geq 2$ are the respective scalar, vector and totally symmetric rank-s' *traceful* tensor fields of the $so(d-1)$ algebra. To simplify the presentation we use oscillators α^i, ζ and collect fields (3.10)-(3.13) into the following ket-vectors:

$$|\phi_s\rangle \equiv \sum_{s'=0}^{s} \frac{\zeta^{s-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-s')!}}\,\phi_s^{i_1\ldots i_{s'}}|0\rangle\,, \tag{3.14}$$

$$|\phi_{s-3}\rangle \equiv \sum_{s'=0}^{s-3} \frac{\zeta^{s-3-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-3-s')!}}\,\phi_{s-3}^{i_1\ldots i_{s'}}|0\rangle\,, \tag{3.15}$$

$$|\mathcal{P}_s\rangle \equiv \sum_{s'=0}^{s} \frac{\zeta^{s-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-s')!}}\,\mathcal{P}_s^{i_1\ldots i_{s'}}|0\rangle\,, \tag{3.16}$$

$$|\mathcal{P}_{s-3}\rangle \equiv \sum_{s'=0}^{s-3} \frac{\zeta^{s-3-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-3-s')!}}\,\mathcal{P}_{s-3}^{i_1\ldots i_{s'}}|0\rangle\,, \tag{3.17}$$

$$|\lambda_{s-1}\rangle \equiv \sum_{s'=0}^{s-1} \frac{\zeta^{s-1-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-1-s')!}}\,\lambda_{s-1}^{i_1\ldots i_{s'}}|0\rangle\,, \tag{3.18}$$

$$|\lambda_{s-2}\rangle \equiv \sum_{s'=0}^{s-2} \frac{\zeta^{s-2-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-2-s')!}}\,\lambda_{s-2}^{i_1\ldots i_{s'}}|0\rangle\,. \tag{3.19}$$

Field content for massless field in AdS_{d+1}. To discuss hamiltonian action for spin-s massless field in AdS space we introduce the following set of fields:

$$\phi_s^{i_1\ldots i_{s'}}\,, \qquad \mathcal{P}_s^{i_1\ldots i_{s'}}\,, \qquad s'=0,1,\ldots,s\,, \tag{3.20}$$

$$\phi_{s-3}^{i_1\ldots i_{s'}}\,, \qquad \mathcal{P}_{s-3}^{i_1\ldots i_{s'}}\,, \qquad s'=0,1,\ldots,s-3\,, \tag{3.21}$$

$$\lambda_{s-1}^{i_1\ldots i_{s'}}\,, \qquad s'=0,1,\ldots,s-1\,, \tag{3.22}$$

$$\lambda_{s-2}^{i_1\ldots i_{s'}}\,, \qquad s'=0,1,\ldots,s-2\,. \tag{3.23}$$

We note that fields in (3.20)-(3.23) with $s'=0$, $s'=1$, and $s'\geq 2$ are the respective scalar, vector and totally symmetric rank-s' *traceful* tensor fields of the $so(d-1)$ algebra. To simplify the

presentation we use oscillators α^i, α^z and collect fields (3.20)-(3.23) into the following ket-vectors:

$$|\phi_s\rangle \equiv \sum_{s'=0}^{s} \frac{\alpha_z^{s-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-s')!}}\,\phi_s^{i_1\ldots i_{s'}}|0\rangle\,, \qquad (3.24)$$

$$|\phi_{s-3}\rangle \equiv \sum_{s'=0}^{s-3} \frac{\alpha_z^{s-3-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-3-s')!}}\,\phi_{s-3}^{i_1\ldots i_{s'}}|0\rangle\,, \qquad (3.25)$$

$$|\mathcal{P}_s\rangle \equiv \sum_{s'=0}^{s} \frac{\alpha_z^{s-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-s')!}}\,\mathcal{P}_s^{i_1\ldots i_{s'}}|0\rangle\,, \qquad (3.26)$$

$$|\mathcal{P}_{s-3}\rangle \equiv \sum_{s'=0}^{s-3} \frac{\alpha_z^{s-3-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-3-s')!}}\,\mathcal{P}_{s-3}^{i_1\ldots i_{s'}}|0\rangle\,, \qquad (3.27)$$

$$|\lambda_{s-1}\rangle \equiv \sum_{s'=0}^{s-1} \frac{\alpha_z^{s-1-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-1-s')!}}\,\lambda_{s-1}^{i_1\ldots i_{s'}}|0\rangle\,, \qquad (3.28)$$

$$|\lambda_{s-2}\rangle \equiv \sum_{s'=0}^{s-2} \frac{\alpha_z^{s-2-s'}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(s-2-s')!}}\,\lambda_{s-2}^{i_1\ldots i_{s'}}|0\rangle\,. \qquad (3.29)$$

Field content for massive field in AdS_{d+1}. To develop hamiltonian approach to spin-s massive field in AdS space we introduce the following set of fields:

$$\phi_{s,n}^{i_1\ldots i_{s'}}\,,\qquad \mathcal{P}_{s,n}^{i_1\ldots i_{s'}}\,,\qquad n\in[s-s']_2\,,\qquad s'=0,1,\ldots,s\,, \qquad (3.30)$$

$$\phi_{s-3,n}^{i_1\ldots i_{s'}}\,,\qquad \mathcal{P}_{s-3,n}^{i_1\ldots i_{s'}}\,,\qquad n\in[s-3-s']_2\,,\qquad s'=0,1,\ldots,s-3\,, \qquad (3.31)$$

$$\lambda_{s-1,n}^{i_1\ldots i_{s'}}\,,\qquad n\in[s-1-s']_2\,,\qquad s'=0,1,\ldots,s-1\,, \qquad (3.32)$$

$$\lambda_{s-2,n}^{i_1\ldots i_{s'}}\,,\qquad n\in[s-2-s']_2\,,\qquad s'=0,1,\ldots,s-2\,. \qquad (3.33)$$

Fields in (3.30)-(3.33) with $s'=0$, $s'=1$, and $s'\geq 2$ are the respective scalar, vector and totally symmetric rank-s' *traceful* tensor fields of the $so(d-1)$ algebra. To simplify the presentation we use oscillators α^i, α_z, ζ and collect fields (3.30)-(3.33) into the following ket-vectors:

$$|\phi_s\rangle = \sum_{s'=0}^{s}\sum_{n\in[s-s']_2} \frac{\zeta^{\frac{s-s'+n}{2}}\alpha_z^{\frac{s-s'-n}{2}}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(\frac{s-s'+n}{2})!(\frac{s-s'-n}{2})!}}\,\phi_{s,n}^{i_1\ldots i_{s'}}|0\rangle\,, \qquad (3.34)$$

$$|\phi_{s-3}\rangle = \sum_{s'=0}^{s-3}\sum_{n\in[s-3-s']_2} \frac{\zeta^{\frac{s-3-s'+n}{2}}\alpha_z^{\frac{s-3-s'-n}{2}}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(\frac{s-3-s'+n}{2})!(\frac{s-3-s'-n}{2})!}}\,\phi_{s-3,n}^{i_1\ldots i_{s'}}|0\rangle\,, \qquad (3.35)$$

$$|\mathcal{P}_s\rangle = \sum_{s'=0}^{s}\sum_{n\in[s-s']_2} \frac{\zeta^{\frac{s-s'+n}{2}}\alpha_z^{\frac{s-s'-n}{2}}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(\frac{s-s'+n}{2})!(\frac{s-s'-n}{2})!}}\,\mathcal{P}_{s,n}^{i_1\ldots i_{s'}}|0\rangle\,, \qquad (3.36)$$

$$|\mathcal{P}_{s-3}\rangle = \sum_{s'=0}^{s-3} \sum_{n\in[s-3-s']_2} \frac{\zeta^{\frac{s-3-s'+n}{2}} \alpha_z^{\frac{s-3-s'-n}{2}} \alpha^{i_1} \ldots \alpha^{i_{s'}}}{s'! \sqrt{(\frac{s-3-s'+n}{2})!(\frac{s-3-s'-n}{2})!}} \, \mathcal{P}_{s-3,n}^{i_1\ldots i_{s'}} |0\rangle \,, \qquad (3.37)$$

$$|\lambda_{s-1}\rangle = \sum_{s'=0}^{s-1} \sum_{n\in[s-1-s']_2} \frac{\zeta^{\frac{s-1-s'+n}{2}} \alpha_z^{\frac{s-1-s'-n}{2}} \alpha^{i_1} \ldots \alpha^{i_{s'}}}{s'! \sqrt{(\frac{s-1-s'+n}{2})!(\frac{s-1-s'-n}{2})!}} \, \lambda_{s-1,n}^{i_1\ldots i_{s'}} |0\rangle \,, \qquad (3.38)$$

$$|\lambda_{s-2}\rangle = \sum_{s'=0}^{s-2} \sum_{n\in[s-2-s']_2} \frac{\zeta^{\frac{s-2-s'+n}{2}} \alpha_z^{\frac{s-2-s'-n}{2}} \alpha^{i_1} \ldots \alpha^{i_{s'}}}{s'! \sqrt{(\frac{s-2-s'+n}{2})!(\frac{s-2-s'-n}{2})!}} \, \lambda_{s-2,n}^{i_1\ldots i_{s'}} |0\rangle \,. \qquad (3.39)$$

To summarize, fields, which we use for discussing the extended hamiltonian approach to massless and massive fields in flat and AdS spaces, can be collected into the following ket-vectors:

$$|\phi_s\rangle \,, \qquad |\phi_{s-3}\rangle \,, \qquad |\mathcal{P}_s\rangle \,, \qquad |\mathcal{P}_{s-3}\rangle \,, \qquad |\lambda_{s-1}\rangle \,, \qquad |\lambda_{s-2}\rangle \,. \qquad (3.40)$$

We note that fields $|\phi_s\rangle$, $|\phi_{s-3}\rangle$, $|\mathcal{P}_s\rangle$, $|\mathcal{P}_{s-3}\rangle$ are phase space variables, while the fields $|\lambda_{s-1}\rangle$, $|\lambda_{s-2}\rangle$ are Lagrange multipliers. In order to obtain the gauge invariant hamiltonian description in easy-to-use form we collect fields (3.40) into 2 vectors given by

$$|\phi\rangle = \left(\begin{array}{c} |\phi_s\rangle \\ |\phi_{s-3}\rangle \end{array}\right) \,, \qquad |\mathcal{P}\rangle = \left(\begin{array}{c} |\mathcal{P}_s\rangle \\ |\mathcal{P}_{s-3}\rangle \end{array}\right) \,, \qquad (3.41)$$

$$|\lambda\rangle = \left(\begin{array}{c} |\lambda_{s-1}\rangle \\ |\lambda_{s-2}\rangle \end{array}\right) \,. \qquad (3.42)$$

Extended hamiltonian action. Extended gauge invariant hamiltonian action for massless and massive fields in flat and AdS spaces takes the form

$$S = \int d^dx \, \mathcal{L} \,, \qquad \text{for flat space} \,, \qquad (3.43)$$

$$S = \int d^dx dz \, \mathcal{L} \,, \qquad \text{for AdS space} \,, \qquad (3.44)$$

where $\mathcal{L}$ is Lagrangian. The Lagrangian we found is given by

$$\mathcal{L} \;\; = \;\; \langle\mathcal{P}|\dot{\phi}\rangle - \frac{1}{2}\langle\mathcal{P}|\mathsf{K}^{-1}|\mathcal{P}\rangle + \langle\mathcal{P}|\mathsf{L}|\phi\rangle + \mathcal{L}^* + \langle\lambda|T\rangle \,, \qquad (3.45)$$

$$\mathcal{L}^* = \frac{1}{2}\langle\phi|E^*|\phi\rangle \,, \qquad (3.46)$$

$$|T\rangle = \bar{G}_\phi|\mathcal{P}\rangle - \bar{G}_{\mathcal{P}}|\phi\rangle \,, \qquad (3.47)$$

where operators constructed out of the spatial derivative ∂^i and the oscillators are given by

$$\Delta_m \equiv \partial^i\partial^i - \mathcal{M}^2 \,, \qquad (3.48)$$

$$\mathsf{K} \equiv \mathsf{K}_0\pi_+ + \mathsf{K}_3\pi_- \,, \qquad (3.49)$$

$$\mathsf{L} \equiv \mathsf{K}_0^{-1}\alpha^2 n_{44} C_{23}\sigma_+ \,, \tag{3.50}$$

$$E^* \equiv \left(n_{00}\Delta_m - C_{10} n_{00}\bar{C}_{10}\right)\pi_+ + \left(\mathsf{K}_3\Delta_m - \bar{C}_{23} n_{44} C_{23} + \bar{C}_{23} n_{44}\bar{\alpha}^2\mathsf{K}_0^{-1}\alpha^2 n_{44} C_{23}\right)\pi_- \,, \tag{3.51}$$

$$\bar{G}_\phi \equiv \bar{G}_{01}\pi_+ + G_{32}\pi_- \,, \tag{3.52}$$

$$\bar{G}_{\mathcal{P}} \equiv G_{31}\sigma_+ + \bar{G}_{02}\sigma_- \,, \tag{3.53}$$

$$C_{10} = \alpha\partial - e_1 \,, \tag{3.54}$$

$$\bar{C}_{10} = \bar{\alpha}\partial - \bar{e}_1 \,, \tag{3.55}$$

$$\bar{C}_{12} = \bar{\alpha}\partial - \bar{e}_1 - e_1\frac{1}{2N_\alpha + d}\bar{\alpha}^2 \,, \tag{3.56}$$

$$C_{23} = \alpha\partial - e_1 - \alpha^2\frac{1}{2N_\alpha + d + 2}\bar{e}_1 \,, \tag{3.57}$$

$$\bar{C}_{23} = \bar{\alpha}\partial - \bar{e}_1 - e_1\frac{1}{2N_\alpha + d + 2}\bar{\alpha}^2 \,, \tag{3.58}$$

$$\bar{G}_{01} = \bar{\alpha}\partial - \bar{e}_1 - e_1\frac{1}{2N_\alpha + d - 2}\bar{\alpha}^2 \,, \tag{3.59}$$

$$G_{12} = \alpha\partial - e_1 - \alpha^2\frac{1}{2N_\alpha + d}\bar{e}_1 \,, \tag{3.60}$$

$$G_{32} = 3\alpha\partial + \alpha^2\bar{\alpha}\partial - e_1(3 + \alpha^2\frac{1}{2N_\alpha + d + 4}\bar{\alpha}^2) - \alpha^2\bar{e}_1 \,, \tag{3.61}$$

$$\bar{G}_{02} \equiv n_{44}\bar{\alpha}^2\Delta_m + \bar{C}_{12} n_{00}\bar{C}_{10} \,, \tag{3.62}$$

$$G_{31} \equiv -G_{21} n_{44} C_{23} \,. \tag{3.63}$$

The 2×2 matrices $\pi_\pm$, $\sigma_\pm$ and operators n_{00}, n_{44}, K_0, K_3 are given in Appendix (see (A.1), (A.12), (A.16), (A.19), (A.20)). We note that the operators n_{00}, n_{44}, K_0, K_3 depend only on the spatial oscillators and are independent of the spatial derivative. From (3.45), we see that the fields $|\phi\rangle$ and $|\mathcal{P}\rangle$ are realized as phase space variables, while the field $|\lambda\rangle$ is realized as Lagrange multiplier.

Gauge transformations. We now discuss realization of gauge symmetries in the framework of hamiltonian gauge invariant approach to massless and massive fields in flat and AdS spaces. We begin our discussion with the description of gauge transformation parameters to be used for description of gauge transformations. We discuss the gauge transformation parameters in turn.

Gauge transformations parameters for massless field in $R^{d-1,1}$. In the framework of our hamiltonian gauge invariant approach, gauge symmetries of spin-s massless field in flat space are described by the following two gauge transformation parameters:

$$\xi^{i_1\ldots i_{s-1}} \,, \qquad \xi^{i_1\ldots i_{s-2}} \,. \tag{3.64}$$

For the corresponding values of s, the gauge transformation parameters in (3.64) are scalar, vector and *traceful* tensor fields of the $so(d-1)$ algebra. We use the oscillators α^i to collect the parameters

into two ket-vectors given by

$$|\xi_{s-1}\rangle \equiv \frac{1}{(s-1)!}\alpha^{i_1}\dots\alpha^{i_{s-1}}\xi^{i_1\dots i_{s-1}}|0\rangle\,, \qquad (3.65)$$

$$|\xi_{s-2}\rangle \equiv \frac{1}{(s-2)!}\alpha^{i_1}\dots\alpha^{i_{s-2}}\xi^{i_1\dots i_{s-2}}|0\rangle\,. \qquad (3.66)$$

Gauge transformations parameters for massive field in $R^{d-1,1}$. In the framework of our hamiltonian gauge invariant approach, gauge symmetries of spin-s massive field in flat space are described by the following set of gauge transformation parameters:

$$\xi_{s-1}^{i_1\dots i_{s'}}\,, \qquad s' = 0,1,\dots,s-1\,, \qquad (3.67)$$

$$\xi_{s-2}^{i_1\dots i_{s'}}\,, \qquad s' = 0,1,\dots,s-2\,. \qquad (3.68)$$

We note that gauge transformation parameters in (3.67),(3.68) with $s'=0$, $s'=1$, and $s'\geq 2$ are the respective scalar, vector, and rank-s' *traceful* tensor fields of the $so(d-1)$ algebra. We use the oscillators α^i, ζ to collect the parameters into two ket-vectors given by

$$|\xi_{s-1}\rangle \equiv \sum_{s'=0}^{s-1}\frac{\zeta^{s-1-s'}\alpha^{i_1}\dots\alpha^{i_{s'}}}{s'!\sqrt{(s-1-s')!}}\,\xi_{s-1}^{i_1\dots i_{s'}}|0\rangle\,, \qquad (3.69)$$

$$|\xi_{s-2}\rangle \equiv \sum_{s'=0}^{s-2}\frac{\zeta^{s-2-s'}\alpha^{i_1}\dots\alpha^{i_{s'}}}{s'!\sqrt{(s-2-s')!}}\,\xi_{s-2}^{i_1\dots i_{s'}}|0\rangle\,. \qquad (3.70)$$

Gauge transformations parameters massless field in AdS_{d+1}. To discuss gauge symmetries of spin-s massless AdS field in the framework of hamiltonian gauge invariant approach, we use the following set of gauge transformation parameters:

$$\xi_{s-1}^{i_1\dots i_{s'}}\,, \qquad s' = 0,1,\dots,s-1\,, \qquad (3.71)$$

$$\xi_{s-2}^{i_1\dots i_{s'}}\,, \qquad s' = 0,1,\dots,s-2\,. \qquad (3.72)$$

Gauge transformation parameters in (3.71),(3.72) with $s'=0$, $s'=1$, and $s'\geq 2$ are the respective scalar, vector, and rank-s' *traceful* tensor fields of the $so(d-1)$ algebra. We use the oscillators α^i, α^z to collect the parameters into two ket-vectors given by

$$|\xi_{s-1}\rangle \equiv \sum_{s'=0}^{s-1}\frac{\alpha_z^{s-1-s'}\alpha^{i_1}\dots\alpha^{i_{s'}}}{s'!\sqrt{(s-1-s')!}}\,\xi_{s-1}^{i_1\dots i_{s'}}|0\rangle\,, \qquad (3.73)$$

$$|\xi_{s-2}\rangle \equiv \sum_{s'=0}^{s-2}\frac{\alpha_z^{s-2-s'}\alpha^{i_1}\dots\alpha^{i_{s'}}}{s'!\sqrt{(s-2-s')!}}\,\xi_{s-2}^{i_1\dots i_{s'}}|0\rangle\,. \qquad (3.74)$$

Gauge transformations parameters for massive field in AdS_{d+1}. To discuss gauge symmetries of spin-s massive AdS field in the framework of hamiltonian gauge invariant approach, we use the following set of gauge transformation parameters:

$$\xi_{s-1,n}^{i_1\dots i_{s'}}\,, \qquad n\in[s-1-s']_2\,, \qquad s' = 0,1,\dots,s-1\,, \qquad (3.75)$$

$$\xi_{s-2,n}^{i_1\ldots i_{s'}}\,, \qquad n\in[s-2-s']_2\,, \qquad s'=0,1,\ldots,s-2\,. \tag{3.76}$$

Gauge transformation parameters in (3.75),(3.76) with $s'=0$, $s'=1$, and $s'\geq 2$ are the respective scalar, vector, and rank-s' *traceful* tensor fields of the $so(d-1)$ algebra. We use the oscillators α^i, α^z, ζ to collect the parameters into two ket-vectors given by

$$|\xi_{s-1}\rangle = \sum_{s'=0}^{s-1}\sum_{n\in[s-1-s']_2} \frac{\zeta^{\frac{s-1-s'+n}{2}}\alpha_z^{\frac{s-1-s'-n}{2}}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(\frac{s-1-s'+n}{2})!(\frac{s-1-s'-n}{2})!}}\,\xi_{s-1,n}^{i_1\ldots i_{s'}}|0\rangle\,, \tag{3.77}$$

$$|\xi_{s-2}\rangle = \sum_{s'=0}^{s-2}\sum_{n\in[s-2-s']_2} \frac{\zeta^{\frac{s-2-s'+n}{2}}\alpha_z^{\frac{s-2-s'-n}{2}}\alpha^{i_1}\ldots\alpha^{i_{s'}}}{s'!\sqrt{(\frac{s-2-s'+n}{2})!(\frac{s-2-s'-n}{2})!}}\,\xi_{s-2,n}^{i_1\ldots i_{s'}}|0\rangle\,. \tag{3.78}$$

To summarize, we note that, in all cases above-considered, the gauge transformation parameters we are going to use for the description of gauge symmetries of massless and massive fields in flat and AdS spaces can be collected into two ket-vectors

$$|\xi_{s-1}\rangle\,, \qquad\qquad |\xi_{s-2}\rangle\,. \tag{3.79}$$

As before, in order to obtain the gauge transformations in easy-to-use form we collect gauge transformation parameters (3.79) into 2 vector given by

$$|\xi\rangle = \left(\begin{array}{c} |\xi_{s-1}\rangle \\ |\xi_{s-2}\rangle \end{array}\right)\,. \tag{3.80}$$

We now ready to discuss gauge transformations. The gauge transformations can entirely be presented in terms of ket-vectors above discussed. The gauge transformations we found take the form

$$\delta|\phi\rangle = G_\phi|\xi\rangle\,, \tag{3.81}$$

$$\delta|\mathcal{P}\rangle = G_{\mathcal{P}}|\xi\rangle\,, \tag{3.82}$$

$$\delta|\lambda\rangle = |\dot\xi\rangle + G_\lambda|\xi\rangle\,, \tag{3.83}$$

$$G_\phi \equiv G_{01}\pi_+ + \bar{G}_{32}\pi_-\,, \tag{3.84}$$

$$G_{\mathcal{P}} \equiv G_{02}\sigma_+ + \bar{G}_{31}\sigma_-\,, \tag{3.85}$$

$$G_\lambda \equiv G_{12}\sigma_+ + \bar{G}_{21}\sigma_-\,, \tag{3.86}$$

$$G_{01} \equiv \alpha\partial - e_1 - \alpha^2\frac{1}{2N_\alpha+d-2}\bar{e}_1\,, \tag{3.87}$$

$$\bar{G}_{32} \equiv 3\bar\alpha\partial + \alpha\partial\bar\alpha^2 - (3+\alpha^2\frac{1}{2N_\alpha+d+4}\bar\alpha^2)\bar{e}_1 - e_1\bar\alpha^2\,, \tag{3.88}$$

$$G_{02} \equiv -\alpha^2 n_{44}\Delta_m - C_{10}n_{00}C_{12}\,, \tag{3.89}$$

$$\bar{G}_{31} \equiv \bar{C}_{23}n_{44}\bar{G}_{21}\,, \tag{3.90}$$

$$G_{12} \equiv \alpha\partial - e_1 - \alpha^2\frac{1}{2N_\alpha+d}\bar{e}_1\,, \tag{3.91}$$

$$\bar{G}_{21} \equiv -\bar{\alpha}\partial + \frac{2N_\alpha + d}{2N_\alpha + d - 2}\bar{e}_1 \,. \qquad (3.92)$$

The following remarks are in order.

i) From (3.83) we see that gauge transformations of the Lagrange multiplier $|\lambda\rangle$ involve time derivative of the gauge transformation parameter, as it should be in extended hamiltonian approach (see e.g. Ref.[2]).

ii) Introducing Hamiltonian H and gauge transformation generating function T_ξ,

$$H \equiv \int d^{d-1}x\, \mathcal{H} \,, \qquad \text{for fields in } R^{d-1,1} \,, \qquad (3.93)$$

$$H \equiv \int d^{d-1}x\, dz\, \mathcal{H} \,, \qquad \text{for fields in } AdS_{d+1} \,, \qquad (3.94)$$

$$-\mathcal{H} \equiv -\frac{1}{2}\langle\mathcal{P}|\mathsf{K}^{-1}|\mathcal{P}\rangle + \langle\mathcal{P}|\mathsf{L}|\phi\rangle + \mathcal{L}^* \,, \qquad (3.95)$$

$$T_\xi \equiv \int d^{d-1}x\, \langle\xi||T\rangle \,, \qquad \text{for fields in } R^{d-1,1} \,, \qquad (3.96)$$

$$T_\xi \equiv \int d^{d-1}x\, dz\, \langle\xi||T\rangle \,, \qquad \text{for fields in } AdS_{d+1} \,, \qquad (3.97)$$

where $|T\rangle$ is given in (3.47), we find that under gauge transformations (3.81)-(3.83) the constraint $|T\rangle$ and Hamiltonian H transform as

$$\delta|T\rangle = 0 \,, \qquad (3.98)$$

$$\delta H = T_{G_\lambda \xi} \,. \qquad (3.99)$$

Relation (3.98) tells that the constraint $|T\rangle$ is invariant under the gauge transformations, while from relation (3.99) we learn that gauge variation of the Hamiltonian H is proportional to the constraint $|T\rangle$. In other words, the $|T\rangle$ is the first-class constraint.

iii) Lagrangian (3.45) implies the standard equal-time Poisson bracket,

$$[|\mathcal{P}\rangle, \langle\phi|] = |\rangle\langle|\delta^{d-1}(x - x') \,, \qquad \text{for fields in } R^{d-1,1} \,, \qquad (3.100)$$

$$[|\mathcal{P}\rangle, \langle\phi|] = |\rangle\langle|\delta^{d-1}(x - x')\delta(z - z') \,, \qquad \text{for fields in } AdS_{d+1} \,, \qquad (3.101)$$

where $|\rangle\langle|$ stands for the unit operator on space of ket-vectors given in (3.41). Using the Poisson bracket, we check that gauge transformations of phase space variables $|\phi\rangle$ and $|\mathcal{P}\rangle$ given in (3.81),(3.82) can be represented as

$$\delta|\phi\rangle = [|\phi\rangle, T_\xi] \,, \qquad (3.102)$$

$$\delta|\mathcal{P}\rangle = [|\mathcal{P}\rangle, T_\xi] \,, \qquad (3.103)$$

as it should be in the framework of the extended hamiltonian approach. Also, in terms of the Poisson bracket, gauge transformations given in (3.98),(3.99) can be represented as

$$[T_{\xi_1}, T_{\xi_2}] = 0 \,, \qquad (3.104)$$

$$[T_\xi, H] = -T_{G_\lambda \xi}\,. \qquad (3.105)$$

iv) As illustration, let us count physical D.o.F for spin-s massless field in d-dimensional flat space by using the extended hamiltonian approach. Using notation $N_{s,n}$ for the dimension of the totally symmetric rank-s traceful tensor field of $so(n)$ algebra,

$$N_{s,n} = \frac{(s+n-1)!}{(n-1)!s!}\,, \qquad (3.106)$$

we note that the dimensions of the fields $\phi^{i_1\ldots i_s}$ and $\phi^{i_1\ldots i_{s-3}}$ are given by $N_{s,d-1}$ and $N_{s-3,d-1}$ respectively, while the dimensions of the Lagrange multipliers $\lambda^{i_1\ldots i_{s-1}}$ and $\lambda^{i_1\ldots i_{s-2}}$ are given by $N_{s-1,d-1}$ and $N_{s-2,d-1}$ respectively. Applying standard formula for counting physical D.o.F (see e.g. Ref.[2]), we find the relation

$$N_{s,d-1} + N_{s-3,d-1} - N_{s-1,d-1} - N_{s-2,d-1} = (2s+d-4)\frac{(s+d-5)!}{(d-4)!s!}\,. \qquad (3.107)$$

Number in r.h.s. in (3.107) is a dimension of totally symmetric rank-s traceless tensor field of $so(d-2)$ algebra. This dimension is the number of physical D.o.F . for spin-s massless field in d-dimensional space-time.

To summarize, staring with the Lagrangian formulation of double-traceless higher-spin fields we obtained the extended hamiltonian action in terms of fields which are free of algebraic constraints. We believe that the appearance of unconstrained fields in the extended hamiltonian approach should streamline application of our approach to the study of various aspects of higher-spin fields.[9] Also, we think that the power of hamiltonian methods will provide new possibilities for analyzing equations of motion of AdS fields and studying AdS/CFT correspondence.[10]

4 Space-time symmetries of extended gauge invariant hamiltonian action

Algebras of space-times symmetries of fields in d-dimensional flat space and fields in AdS_{d+1} space contain the Lorentz subalgebra $so(d-1,1)$. In the framework of the hamiltonian approach, the Lorentz symmetries $so(d-1,1)$ are not realized manifestly. Therefore, in the framework of the hamiltonian approach, complete description of field dynamics implies that we have to work out explicit realization of space-time symmetries. We now discuss the realization of space-time symmetries in the framework of our approach.

In the hamiltonian approach, the Poincaré algebra generators can be separated into two groups:

$$P^i, \quad J^{ij}, \qquad \text{kinematical generators}\,; \qquad (4.1)$$

$$P^0, \quad J^{0i}\,, \qquad \text{dynamical generators}\,. \qquad (4.2)$$

To discuss AdS space-time symmetries the Poincaré symmetries should be supplemented by the dilatation symmetry and conformal boost symmetries which also can be separated into two groups,

$$K^i, \quad D, \qquad \text{kinematical generators}\,; \qquad (4.3)$$

[9] As a side remark we note that at Lagrangian level many interesting formulations in terms of unconstrained fields were developed in last years. This is to say various Lagrangian formulations of higher-spin field dynamics in terms of unconstrained fields are discussed in Refs.[44]-[48].

[10] Discussion of interesting methods for analyzing equations of motion of fields in AdS space may be found in Refs.[49]-[51].

$$K^0\,, \qquad\qquad \text{dynamical generators}\,. \tag{4.4}$$

For $t = 0$, the kinematical generators in the field realization are quadratic in the physical fields

Lie derivative realization of kinematical Poincaré symmetries. To simplify our formulas let us use the following shortcuts for the ket-vectors:

$$\phi \equiv |\phi\rangle\,, \quad \mathcal{P} \equiv |\mathcal{P}\rangle\,, \quad \lambda \equiv |\lambda\rangle\,. \tag{4.5}$$

Using the notation

$$\chi = \phi\,, \quad \mathcal{P}\,, \quad \lambda\,, \tag{4.6}$$

we note that the realization of spatial translations and the $so(d-1)$ algebra transformations take the standard well-known form,

$$[\chi, P^i] = \partial^i \chi\,, \tag{4.7}$$

$$[\chi, J^{ij}] = (x^i\partial^j - x^j\partial^i + M^{ij})\chi\,, \tag{4.8}$$

$$M^{ij} \equiv \alpha^i\bar{\alpha}^j - \alpha^j\bar{\alpha}^i\,. \tag{4.9}$$

Lie derivative realization of dynamical Poincaré symmetries. Using the notation as in (4.6) we note that P^0 transformation takes the standard form

$$[\chi, P^0] = -\dot{\chi}\,. \tag{4.10}$$

Lorentz boost J^{0i} transformations of fields ϕ, $\mathcal{P}$, and λ are found to be

$$[\phi, J^{0i}] = l^{0i}\phi + M^{0i}_{\phi\phi}\phi + M^{0i}_{\phi\lambda}\lambda\,, \tag{4.11}$$

$$[\lambda, J^{0i}] = l^{0i}\lambda + M^{0i}_{\lambda\phi}\phi + M^{0i}_{\lambda\lambda}\lambda\,, \tag{4.12}$$

$$[\mathcal{P}, J^{0i}] = l^{0i}\mathcal{P} + M^{0i}_{\mathcal{P}\mathcal{P}}\mathcal{P} + M^{0i}_{\mathcal{P}\phi}\phi + M^{0i}_{\mathcal{P}\lambda}\lambda\,, \tag{4.13}$$

$$l^{0i} \equiv t\partial^i + x^i\partial_t\,. \tag{4.14}$$

We note that spin operators $M^{0i}_{\phi\phi}$, $M^{0i}_{\phi\lambda}$, $M^{0i}_{\lambda\phi}$, $M^{0i}_{\lambda\lambda}$, and $M^{0i}_{\mathcal{P}\mathcal{P}}$ depend only on the oscillators, while spin operators $M^{0i}_{\mathcal{P}\phi}$, $M^{0i}_{\mathcal{P}\lambda}$ depend on the oscillators and the spatial derivative. Explicit form of the spin operators is given in Appendix (see (A.43)-(A.62)).

Poincaré algebra transformations above-described are valid for fields in flat and AdS spaces. For fields in AdS space, to complete description of space-time symmetries we discuss field transformations under dilatation and conformal boost symmetries.

Lie derivative realization of dilatation and conformal boost transformations. Transformations of fields ϕ, $\mathcal{P}$, and λ under dilatation symmetry take the form

$$[\phi, D] = (t\partial_t + x\partial + \Delta)\phi\,, \tag{4.15}$$

$$[\mathcal{P}, D] = (t\partial_t + x\partial + \Delta + 1)\mathcal{P}\,, \tag{4.16}$$

$$[\lambda, D] = (t\partial_t + x\partial + \Delta)\lambda\,, \tag{4.17}$$

$$x\partial \equiv x^i\partial^i\,, \qquad \Delta \equiv z\partial_z + \frac{d-1}{2}\,. \tag{4.18}$$

Conformal boost transformations of ϕ and λ take the form

$$[\phi, K^i] = (\frac{1}{2}t^2\partial^i + tx^i\partial_t)\phi + tM^{0i}_{\phi\phi}\phi + tM^{0i}_{\phi\lambda}\lambda + k^i_{\Delta,M}\phi + R^i_{\phi\phi}\phi\,, \tag{4.19}$$

$$[\lambda, K^i] = (\frac{1}{2}t^2\partial^i + tx^i\partial_t)\lambda + tM^{0i}_{\lambda\phi}\phi + tM^{0i}_{\lambda\lambda}\lambda + k^i_{\Delta,M}\lambda + R^i_{\lambda\phi}\phi + R^i_{\lambda\lambda}\lambda\,, \tag{4.20}$$

$$k^i_{\Delta,M} \equiv -\frac{1}{2}x^jx^j\partial^i + x^i(x\partial + \Delta) + M^{ij}x^j\,. \tag{4.21}$$

We note that operators $R^i_{\phi\phi}$, $R^i_{\lambda\phi}$, $R^i_{\lambda\lambda}$ depend on the oscillators, on the radial AdS coordinate z, and the spatial derivative. Explicit form of these operators is given in Appendix (see (A.63)-(A.70)).

Canonical realization of dynamical Poincaré symmetries. Using Lie derivative realization given in (4.10) and the standard procedure, we obtain the canonical realization of P^0 transformations,

$$[\phi, P^0] = [\phi, H]\,, \tag{4.22}$$

$$[\mathcal{P}, P^0] = [\mathcal{P}, H]\,, \tag{4.23}$$

$$[\lambda, P^0] = G_\lambda\lambda\,. \tag{4.24}$$

Using Hamiltonian H given in (3.93)-(3.95) and Poisson brackets in (3.100),(3.101), transformation rules of phase space variables in (4.22),(4.23) can be represented as

$$[\phi, P^0] = -\mathsf{K}^{-1}\mathcal{P} + \mathsf{L}\phi\,, \tag{4.25}$$

$$[\mathcal{P}, P^0] = \bar{\mathsf{L}}\mathcal{P} - E^*\phi\,, \qquad \bar{\mathsf{L}} \equiv -\mathsf{L}^\dagger\,. \tag{4.26}$$

Canonical realization of J^{0i} transformations is given by

$$[\phi, J^{0i}] = t\partial^i\phi - x^i[\phi, H] + M^{0i}_{\phi\phi}\phi\,, \tag{4.27}$$

$$[\mathcal{P}, J^{0i}] = t\partial^i\mathcal{P} - x^i[\mathcal{P}, H] + M^{0i}_{\mathcal{P}\mathcal{P}}\mathcal{P} + M^{0i}_{\mathcal{P}\phi}\phi\,, \tag{4.28}$$

$$[\lambda, J^{0i}] = t\partial^i\lambda - G_\lambda x^i\lambda + M^{0i}_{\lambda\phi}\phi + M^{0i}_{\lambda\lambda}\lambda\,. \tag{4.29}$$

Using Hamiltonian H given in (3.93)-(3.95) and Poisson brackets in (3.100),(3.101), transformation rules of phase space variables in (4.27),(4.28) can explicitly be represented as

$$[\phi, J^{0i}] = t\partial^i\phi + x^i\mathsf{K}^{-1}\mathcal{P} - x^i\mathsf{L}\phi + M^{0i}_{\phi\phi}\phi\,, \tag{4.30}$$

$$[\mathcal{P}, J^{0i}] = t\partial^i\mathcal{P} - x^i\bar{\mathsf{L}}\mathcal{P} + x^iE^*\phi + M^{0i}_{\mathcal{P}\mathcal{P}}\mathcal{P} + M^{0i}_{\mathcal{P}\phi}\phi\,. \tag{4.31}$$

Canonical realization of Poincaré algebra transformations above-described is valid for fields in flat and AdS spaces. For fields in AdS space, to complete description of space-symmetries we now present the field transformations under the dilatation and conformal boost symmetries.

Canonical realization of dilatation transformations.

$$[\phi, D] = -t[\phi, H] + (x\partial + \Delta)\phi\,, \tag{4.32}$$

$$[\mathcal{P}, D] = -t[\mathcal{P}, H] + (x\partial + \Delta + 1)\mathcal{P}\,, \tag{4.33}$$

$$[\lambda, D] = -tG_\lambda\lambda + (x\partial + \Delta - 1)\lambda\,. \tag{4.34}$$

Canonical realization of conformal boost transformations.

$$[\phi, K^i] = \frac{1}{2}t^2\partial^i\phi - t[\phi, H] + tM^{0i}_{\phi\phi}\phi + tM^{0i}_{\phi\lambda}\lambda + k^i_{\Delta,M}\phi + R^i_{\phi\phi}\phi\,, \tag{4.35}$$

$$[\lambda, K^i] = \frac{1}{2}t^2\partial^i\lambda - tG_\lambda(x^i\lambda) + tM^{0i}_{\lambda\phi}\phi + tM^{0i}_{\lambda\lambda}\lambda + k^i_{\Delta-1,M}\lambda + R^i_{\lambda\phi}\phi + R^i_{\lambda\lambda}\lambda\,. \tag{4.36}$$

In conclusion, we note a number of the potentially interesting generalizations and applications of our approach. This is to say that although many methods for building interaction vertices for higher-spin fields are known in the literature (see e.g. Refs.[52]-[66]), constructing interaction vertices for concrete field theoretical models of higher-spin fields is still a challenging problem. We believe that use of the extended hamiltonian approach will provide new interesting possibilities for studying this important problem. Also we think that the extended hamiltonian approach we discussed in this paper might be useful for the study of string theory in AdS background [67]-[69] and various aspects of AdS/CFT correspondence along the lines in Refs.[70]-[73]. In this paper we considered the extended hamiltonian action for the bosonic totally symmetric fields. Needless to say that generalization of our approach to the case of fermionic fields [74] and mixed symmetry fields [75]-[80] could also be of interest.

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Appendix A Notation

Basis of 2×2 we use is defined as

$$\sigma_+ = \left(\begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array}\right), \quad \sigma_- = \left(\begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array}\right), \quad \pi_+ = \left(\begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array}\right), \quad \pi_- = \left(\begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array}\right). \tag{A.1}$$

Throughout the paper the notation $n\in[k]_2$ implies that $n=-k,-k+2,-k+4,\ldots,k-4,k-2,k$:

$$n\in[k]_2 \quad\Longrightarrow\quad n=-k,-k+2,-k+4,\ldots,k-4,k-2,k\,. \tag{A.2}$$

Notation in basis of Lorentz algebra $so(d-1,1)$. Our conventions are as follows. x^a denotes coordinates in d-dimensional flat space-time, while ∂_a denotes derivatives with respect to x^a, $\partial_a \equiv \partial/\partial x^a$. Vector indices of the Lorentz algebra $so(d-1,1)$ take the values $a,b,c,e=0,1,\ldots,d-1$. We use the mostly positive flat metric tensor η^{ab}. To simplify our expressions we drop η_{ab} in scalar products, i.e., we use $X^aY^a \equiv \eta_{ab}X^aY^b$.

We use a set of the creation operators α^a, α^z, ζ, and the respective set of annihilation operators $\bar\alpha^a$, $\bar\alpha^z$, $\bar\zeta$. These operators, to be referred to as oscillators, satisfy the commutation relations[11]

$$[\bar\alpha^a, \alpha^b] = \eta^{ab}\,, \qquad [\bar\zeta, \zeta] = 1\,, \qquad [\bar\alpha^z, \alpha^z] = 1\,, \tag{A.3}$$

$$\bar\alpha^a|0\rangle = 0\,, \qquad \bar\alpha^z|0\rangle = 0\,, \qquad \bar\zeta|0\rangle = 0\,. \tag{A.4}$$

We adapt the following hermitian conjugation rules for the derivatives and oscillators:

$$\partial^{a\dagger} = -\partial^a\,, \qquad \alpha^{a\dagger} = \bar\alpha^a\,, \qquad \alpha^{z\dagger} = \bar\alpha^z\,, \qquad \zeta^\dagger = \bar\zeta\,. \tag{A.5}$$

[11] Extensive study and applications of the oscillator formalism may be fond in Refs.[81, 82].

We use operators constructed out of the derivatives and oscillators,

$$\Box = \partial^a\partial^a\,, \qquad \boldsymbol{\alpha\partial} \equiv \alpha^a\partial^a\,, \qquad \bar{\boldsymbol{\alpha}}\boldsymbol{\partial} \equiv \bar{\alpha}^a\partial^a\,, \tag{A.6}$$

$$\boldsymbol{\alpha}^2 \equiv \alpha^a\alpha^a\,, \qquad \bar{\boldsymbol{\alpha}}^2 \equiv \bar{\alpha}^a\bar{\alpha}^a\,, \qquad N_{\boldsymbol{\alpha}} \equiv \alpha^a\bar{\alpha}^a\,, \tag{A.7}$$

$$N_z \equiv \alpha^z\bar{\alpha}^z\,, \qquad N_\zeta \equiv \zeta\bar{\zeta}\,. \tag{A.8}$$

Notation in basis of $so(d-1)$ **algebra**. In the basis of $so(d-1)$ algebra, we split the space-time coordinates, derivatives, and oscillators as follows

$$x^a = t, x^i\,, \qquad \partial_a = \partial_t, \partial_i\,, \qquad \partial_t \equiv \partial/\partial t\,, \qquad \partial_i \equiv \partial/\partial x^i\,, \tag{A.9}$$

$$\alpha^a = \alpha^0\,, \alpha^i\,, \qquad \bar{\alpha}^a = \bar{\alpha}^0\,, \bar{\alpha}^i\,, \qquad [\bar{\alpha}^0, \alpha^0] = -1\,, \qquad [\bar{\alpha}^i, \alpha^j] = \delta^{ij}\,. \tag{A.10}$$

Vector indices of the algebra $so(d-1)$ take the values $i,j = 1,\ldots,d-1$. We use operators constructed out of the spatial derivative and oscillators,

$$\alpha\partial \equiv \alpha^i\partial^i\,, \qquad \bar{\alpha}\partial \equiv \bar{\alpha}^i\partial^i\,, \qquad \alpha^2 \equiv \alpha^i\alpha^i\,, \qquad \bar{\alpha}^2 \equiv \bar{\alpha}^i\bar{\alpha}^i\,, \qquad N_\alpha \equiv \alpha^i\bar{\alpha}^i\,, \tag{A.11}$$

$$n_{00} = \sum_{n=0}^\infty \frac{1}{(2n)!}\alpha^{2n}\bar{\alpha}^{2n}\,, \tag{A.12}$$

$$n_{11} = -\sum_{n=0}^\infty \frac{2n+1}{(2n)!}\alpha^{2n}\bar{\alpha}^{2n}\,, \tag{A.13}$$

$$n_{22} = \sum_{n=0}^\infty \frac{2n+2}{(2n+1)!}\alpha^{2n}\bar{\alpha}^{2n}\,, \tag{A.14}$$

$$n_{33} = -\sum_{n=0}^\infty \frac{4(n+1)^2}{(2n+3)!}\alpha^{2n}\bar{\alpha}^{2n}\,. \tag{A.15}$$

$$n_{44} = -\sum_{n=0}^\infty \frac{1}{(2n+1)!}\alpha^{2n}\bar{\alpha}^{2n}\,, \tag{A.16}$$

$$n_{02} = \alpha^2\sum_{n=0}^\infty \frac{1}{(2n+1)!}\alpha^{2n}\bar{\alpha}^{2n}\,, \tag{A.17}$$

$$n_{13} = \alpha^2\sum_{n=0}^\infty \frac{1}{(2n+1)!}\alpha^{2n}\bar{\alpha}^{2n}\,, \tag{A.18}$$

$$\mathsf{K}_0 = \sum_{n=0}^\infty \frac{1-2n}{(2n)!}\alpha^{2n}\bar{\alpha}^{2n}\,, \tag{A.19}$$

$$\mathsf{K}_3 = \sum_{n=0}^\infty \frac{2n+2}{(2n+3)!}\alpha^{2n}\bar{\alpha}^{2n}\,, \tag{A.20}$$

$$\mathsf{K}_0 \equiv n_{00} + \alpha^2 n_{44}\bar{\alpha}^2\,, \tag{A.21}$$

$$\mathsf{K}_3 \equiv n_{33} - n_{44}\,, \tag{A.22}$$

$$G_{01} = \alpha\partial - e_1 - \alpha^2 \frac{1}{2N_\alpha + d - 2}\bar{e}_1\,, \tag{A.23}$$

$$\bar{G}_{01} = \bar{\alpha}\partial - \bar{e}_1 - e_1 \frac{1}{2N_\alpha + d - 2}\bar{\alpha}^2\,, \tag{A.24}$$

$$G_{12} = \alpha\partial - e_1 - \alpha^2 \frac{1}{2N_\alpha + d}\bar{e}_1\,, \tag{A.25}$$

$$\bar{G}_{12} = \bar{\alpha}\partial - \bar{e}_1 - e_1 \frac{1}{2N_\alpha + d}\bar{\alpha}^2\,, \tag{A.26}$$

$$G_{21} = -\alpha\partial + e_1 \frac{2N_\alpha + d}{2N_\alpha + d - 2}\,, \tag{A.27}$$

$$\bar{G}_{21} = -\bar{\alpha}\partial + \frac{2N_\alpha + d}{2N_\alpha + d - 2}\bar{e}_1\,, \tag{A.28}$$

$$G_{32} = 3\alpha\partial + \alpha^2\bar{\alpha}\partial - e_1(3 + \alpha^2 \frac{1}{2N_\alpha + d + 4}\bar{\alpha}^2) - \alpha^2\bar{e}_1\,, \tag{A.29}$$

$$\bar{G}_{32} = 3\bar{\alpha}\partial + \alpha\partial\bar{\alpha}^2 - (3 + \alpha^2 \frac{1}{2N_\alpha + d + 4}\bar{\alpha}^2)\bar{e}_1 - e_1\bar{\alpha}^2\,, \tag{A.30}$$

$$G_{02} \equiv -\alpha^2 n_{44}\Delta_m - C_{10}n_{00}C_{12}\,, \tag{A.31}$$

$$\bar{G}_{02} \equiv n_{44}\bar{\alpha}^2\Delta_m + \bar{C}_{12}n_{00}\bar{C}_{10}\,, \tag{A.32}$$

$$G_{31} \equiv -G_{21}n_{44}C_{23}\,, \tag{A.33}$$

$$\bar{G}_{31} \equiv \bar{C}_{23}n_{44}\bar{G}_{21}\,, \tag{A.34}$$

$$C_{10} = \alpha\partial - e_1\,, \tag{A.35}$$

$$\bar{C}_{10} = \bar{\alpha}\partial - \bar{e}_1\,, \tag{A.36}$$

$$C_{12} = \alpha\partial - e_1 - \alpha^2 \frac{1}{2N_\alpha + d}\bar{e}_1\,, \tag{A.37}$$

$$\bar{C}_{12} = \bar{\alpha}\partial - \bar{e}_1 - e_1 \frac{1}{2N_\alpha + d}\bar{\alpha}^2\,, \tag{A.38}$$

$$C_{21} = \alpha\partial + \alpha^2\bar{\alpha}\partial - (1 + \alpha^2 \frac{1}{2N_\alpha + d + 2}\bar{\alpha}^2)e_1 - \alpha^2\bar{e}_1\,, \tag{A.39}$$

$$\bar{C}_{21} = \bar{\alpha}\partial + \alpha\partial\bar{\alpha}^2 - (1 + \alpha^2 \frac{1}{2N_\alpha + d + 2}\bar{\alpha}^2)\bar{e}_1 - e_1\bar{\alpha}^2\,, \tag{A.40}$$

$$C_{23} = \alpha\partial - e_1 - \alpha^2 \frac{1}{2N_\alpha + d + 2}\bar{e}_1\,, \tag{A.41}$$

$$\bar{C}_{23} = \bar{\alpha}\partial - \bar{e}_1 - e_1 \frac{1}{2N_\alpha + d + 2}\bar{\alpha}^2 \,, \tag{A.42}$$

$$M^{0i}_{\phi\phi} = \sigma_- M^{0i}_{\phi_{s-3}\phi_s} \,, \tag{A.43}$$

$$M^{0i}_{\phi\lambda} = \pi_+ M^{0i}_{\phi_s\lambda_{s-1}} + \pi_- M^{0i}_{\phi_{s-3}\lambda_{s-2}} \,, \tag{A.44}$$

$$M^{0i}_{\lambda\phi} = \pi_+ M^{0i}_{\lambda_{s-1}\phi_s} + \pi_- M^{0i}_{\lambda_{s-2}\phi_{s-3}} \,, \tag{A.45}$$

$$M^{0i}_{\lambda\lambda} = \sigma_+ M^{0i}_{\lambda_{s-1}\lambda_{s-2}} + \sigma_- M^{0i}_{\lambda_{s-2}\lambda_{s-1}} \,. \tag{A.46}$$

$$M^{0i}_{\mathcal{P}\mathcal{P}} = \sigma_+ M^{0i}_{\mathcal{P}_s\mathcal{P}_{s-3}} + \sigma_- M^{0i}_{\mathcal{P}_{s-3}\mathcal{P}_s} \,, \tag{A.47}$$

$$M^{0i}_{\mathcal{P}\phi} = \pi_+ M^{0i}_{\mathcal{P}_s\phi_s} + \pi_- M^{0i}_{\mathcal{P}_{s-3}\phi_{s-3}} \,, \tag{A.48}$$

$$M^{0i}_{\mathcal{P}\lambda} = \sigma_+ M^{0i}_{\mathcal{P}_s\lambda_{s-2}} + \sigma_- M^{0i}_{\mathcal{P}_{s-3}\lambda_{s-1}} \,, \tag{A.49}$$

$$M^{0i}_{\phi_{s-3}\phi_s} = (3\bar{\alpha}^i + \alpha^i\bar{\alpha}^2)\bar{\alpha}^2 \,, \tag{A.50}$$

$$M^{0i}_{\phi_s\lambda_{s-1}} = \alpha^i \,, \tag{A.51}$$

$$M^{0i}_{\phi_{s-3}\lambda_{s-2}} = 3\bar{\alpha}^i + \alpha^i\bar{\alpha}^2 \,, \tag{A.52}$$

$$M^{0i}_{\lambda_{s-1}\phi_s} = \bar{\alpha}^i + \alpha^i\bar{\alpha}^2 \,, \tag{A.53}$$

$$M^{0i}_{\lambda_{s-2}\phi_{s-3}} = -\alpha^i \,, \tag{A.54}$$

$$M^{0i}_{\lambda_{s-1}\lambda_{s-2}} = 2\alpha^i \,, \tag{A.55}$$

$$M^{0i}_{\lambda_{s-2}\lambda_{s-1}} = \alpha^i\bar{\alpha}^2 \,, \tag{A.56}$$

$$M^{0i}_{\mathcal{P}_s\mathcal{P}_{s-3}} \equiv -\alpha^2(3\alpha^i + \alpha^2\bar{\alpha}^i) \,, \tag{A.57}$$

$$M^{0i}_{\mathcal{P}_s\phi_s} \equiv n_{00}\partial^i - C_{10}n_{00}(\bar{\alpha}^i + \alpha^i\bar{\alpha}^2) + \alpha^2\bar{\alpha}^i n_{00}\bar{C}_{10} \,, \tag{A.58}$$

$$M^{0i}_{\mathcal{P}_s\lambda_{s-2}} \equiv -2\alpha^2 n_{44}\partial^i - \alpha^i n_{00}C_{12} - C_{10}n_{00}\alpha^i \,, \tag{A.59}$$

$$M^{0i}_{\mathcal{P}_{s-3}\mathcal{P}_s} \equiv -\bar{\alpha}^i n_{44}\bar{\alpha}^2\mathsf{K}_0^{-1} \,, \tag{A.60}$$

$$M^{0i}_{\mathcal{P}_{s-3}\lambda_{s-1}} \equiv \bar{\alpha}^i n_{44}\bar{G}_{21} - \bar{C}_{23}n_{44}\bar{\alpha}^i \,, \tag{A.61}$$

$$M^{0i}_{\mathcal{P}_{s-3}\phi_{s-3}} \equiv \mathsf{K}_3\partial^i - \bar{C}_{23}n_{44}\alpha^i + \bar{\alpha}^i n_{44}\bar{\alpha}^2\mathsf{K}_0^{-1}\alpha^2 n_{44}C_{23} \,, \tag{A.62}$$

$$R^i_{\phi\phi} = R^i_{\phi_s\phi_s}\pi_+ + R_{\phi^i_{s-3}\phi_{s-3}}\pi_- \,, \tag{A.63}$$

$$R^i_{\lambda\phi} = R^i_{\lambda_{s-2}\phi_s}\sigma_- \,, \tag{A.64}$$

$$R^i_{\lambda\lambda} = R_{\lambda^i_{s-1}\lambda_{s-1}}\pi_+ + R^i_{\lambda_{s-2}\lambda_{s-2}}\pi_- \,, \tag{A.65}$$

$$R^i_{\phi_s\phi_s} = r\bar\alpha^i + (\alpha^i - \alpha^2\frac{1}{2N_\alpha + d - 2}\bar\alpha^i)\bar r - \frac{1}{2}z^2\partial^i \,, \tag{A.66}$$

$$R^i_{\lambda_{s-1}\lambda_{s-1}} = r\bar\alpha^i + (\alpha^i - \alpha^2\frac{1}{2N_\alpha + d}\bar\alpha^i)\bar r - \frac{1}{2}z^2\partial^i \,, \tag{A.67}$$

$$R^i_{\lambda_{s-2}\lambda_{s-2}} = r\bar\alpha^i + (\alpha^i - \alpha^2\frac{1}{2N_\alpha + d + 2}\bar\alpha^i)\bar r - \frac{1}{2}z^2\partial^i \,, \tag{A.68}$$

$$R^i_{\lambda_{s-2}\phi_s} = \frac{2}{2N_\alpha + d - 2}\bar\alpha^i\bar r \,, \tag{A.69}$$

$$R^i_{\phi_{s-3}\phi_{s-3}} = r\bar\alpha^i + (\alpha^i - \alpha^2\frac{1}{2N_\alpha + d + 4}\bar\alpha^i)\bar r - \frac{1}{2}z^2\partial^i \,. \tag{A.70}$$

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